nature neuroscience

Human representation of visuo-motor uncertainty as mixtures of orthogonal basis distributions

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In many laboratory visuo-motor decision tasks, subjects compensate for their own visuo-motor error, earning close to the maximum reward possible. To do so, they must combine information about the distribution of possible error with values associated with different movement outcomes. The optimal solution is a potentially difficult computation that presupposes knowledge of the probability density function (pdf) of visuo-motor error associated with each possible planned movement. It is unclear how the brain represents such pdfs or computes with them. In three experiments, we used a forced-choice method to reveal subjects' internal representations of their spatial visuo-motor error in a speeded reaching movement. Although subjects' objective distributions were unimodal, close to Gaussian, their estimated internal pdfs were typically multimodal and were better described as mixtures of a small number of distributions differing only in location and scale. Mixtures of a small number of uniform distributions outperformed other mixture distributions, including mixtures of Gaussians.

... [1–3] ... [4,11] ... [12,13] ... [14] ... (Fig. 1a) ... $G(x)$, ... a, ... $EG = \int G(x) f(x - a)dx$, ... [2] ... Figure 1b, ...

... $b_1(x), \ldots, b_n(x)$ ... [15]

$$f(x) \approx \sum_{i=1}^{n} w_i b_i(x) \tag{1}$$

... Figure 1a, ... (Fig. 1a) ... 6.

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Received 18 March; accepted 5 June; published online 29 June 2015; doi:10.1038/nn.4055

Figure 2 ... 1. ... (Fig. 2a) ... (Fig. 2b). ... (Fig. 2c). ... Fig. 2d ...

... [17]. ... (Fig. 2e).

RESULTS

Experiment 1: objective pdf

... (Fig. 2b) ... (0.04 1.78, ... 0.44), ... t ... t ... $r = 1.0$, $P < 0.001$) ... $r = 0.82$, $P = 0.004$). ... t ...

Experiment 1: internal pdf

...

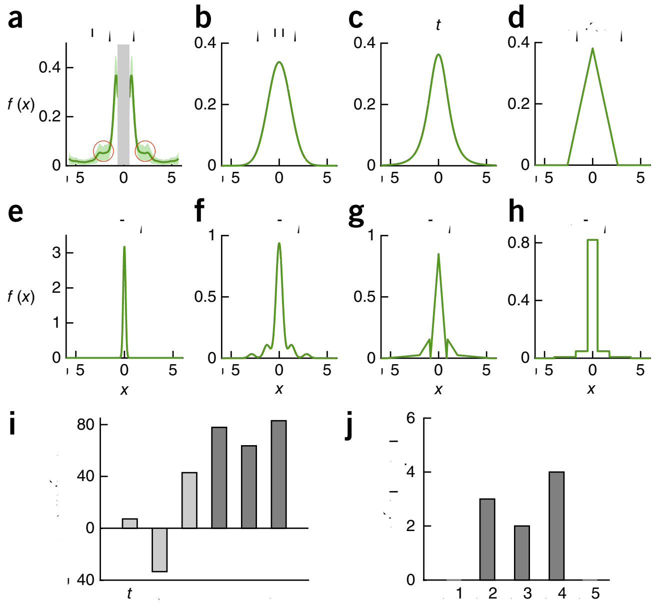


Figure 3 Internal pdfs in the choice task of experiment 1. (**a**) Non-parametric visualization of the internal pdf for one subject. Green-shaded regions denote ±s.e.m. *x* is in the unit of the subject's horizontal s.d. estimated from the reaching task. The gray-shaded central range of [–0.6, 0.6] could not be reliably estimated in experiment 1 (Online Methods) and the visualization therefore gives information about the pdf only away from the origin. Two regions of interest are marked by red circles. The visualizations for all subjects are shown in **Supplementary Figure 1**. (**b**–**h**) Internal pdfs estimated from different models for the same subject. (**i**) AICc difference between the Gaussian model and the other six models summed over the nine subjects. The unimodal models (including vG-mix) and mixture models are coded in light gray and dark gray, respectively. Positive difference indicates better fit. LD denotes linear decay. (**j**) Number of subjects best fit by each U-mix model.

(A C)[18,19]

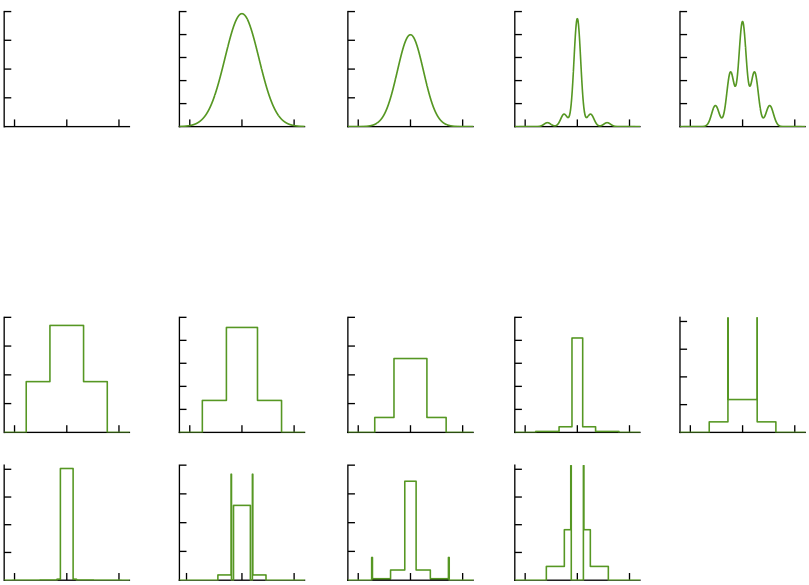
Unimodal distributions.

Mixture distributions.

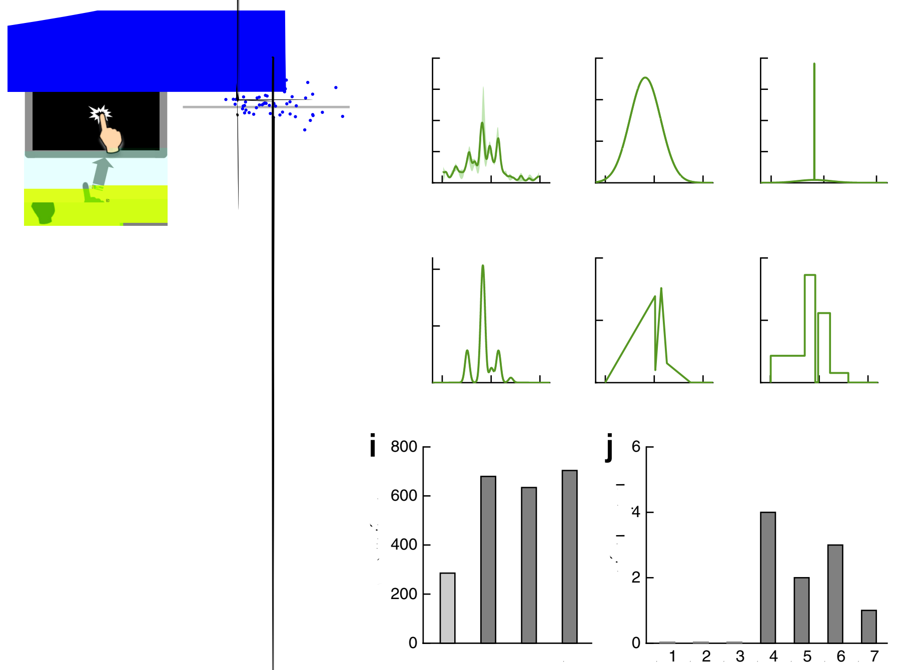
Figure 1,

Figure 3b h.

Figure 3i.



(Supplementary Fig. 2).



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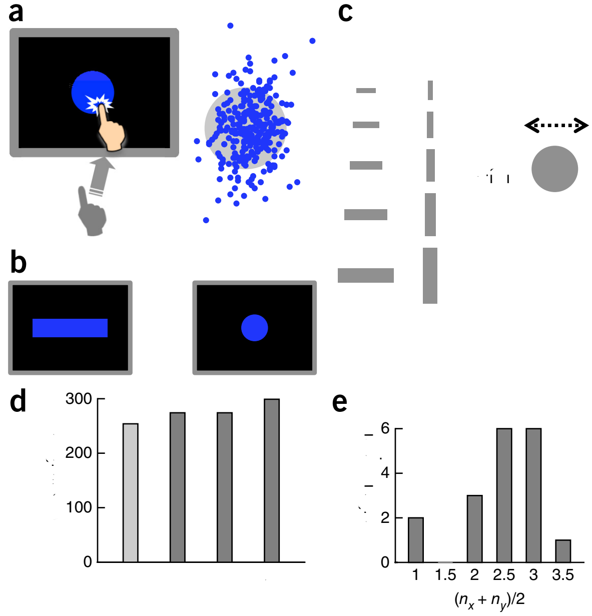

Figure 6 Experiment 3. (**a**) The reaching task. Left, the task. The task for experiment 3 was the same as that for experiment 1, except that a circular target was used. Right, the endpoints for one subject. (**b**) The choice task. The task for experiment 3 was the same as that for experiment 1, except that each pair of targets was a rectangle and a circle. (**c**) Design of the choice task. Ten different rectangles were used; for each, the radius of its paired circle was adjusted by adaptive procedures for 100 trials. (**d**) AICc difference between the Gaussian model and the other four models summed over the 18 subjects. The unimodal models (including vG-mix) and mixture models are coded in light gray and dark gray, respectively. Positive difference indicates better fit. (**e**) Number of subjects best fit by each U-mix model.

DISCUSSION

Relationship to previous measures

Discrete representation and near-optimal motor decisions

Simplifying probabilistic calculation

Discrete representation as explanation for decision biases

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$b_i > 0, h_i$

Summary of statistical tests. 21. A C[18,19] B[20]

A **Supplementary Methods Checklist**

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